Training teachers to manage problem-solving classes with computer support

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Resumo: O uso limitado de tecnologias nas salas de aula pode ser conseqüência da sua falta nas escolas, da despreocupação dos professores sobre os recursos disponíveis e da necessidade de adaptá-las às situações das aulas; ou da insuficiência teórica e prática para que os professores as integrem ao seu ensino. Neste artigo propomos formas e descrevemos exemplos do uso do GeogebraTUTOR, um tutorial que incorpora elementos da geometria dinâmica, para um treinamento inicial focalizado na resolução de problemas, raciocínio matemático e processo de comunicação. Adicionalmente, discutimos a necessidade de habilidades profissionais relativas ao processo de resolução de problemas para o ensino e aprendizado com suporte computacional em sala de aula.

Palavras-chave: tecnologias na sala de aula, treinamento docente, resolução de problemas.

Abstract: The limited use of technology in classrooms may be a consequence of a lack of technological means in schools, of teachers' unawareness of available ICT teaching resources and the need of having to adapt them to their classroom situation; or of the insufficient theoretical and practical training that teachers have to allow them to successfully integrate technology learning environments into their teaching. In this paper we propose ways and describe examples of using GeogebraTUTOR, a tutorial system incorporating elements of dynamic geometry, for initial teacher training focusing on problem solving, mathematical reasoning, and communication processes. In addition, we discuss the necessary professional skills of teachers related to the processes of teaching and learning problem solving with computer support in the classroom.

Keywords: technology in classrooms, teacher training, problem solving.

1 INTRODUCTION

The contrast between the non-technological world in which young people live in their school environment and the technological world by which they are surrounded in their everyday life (mobile telephones, messaging programmes, sound and video equipment...) is so large that we should reflect on the causes, and ways of overcoming these, of the prevalent absence of technology use in mathematics teaching.

Little use of technology in mathematics classrooms may be a consequence of a lack of technological means, which are often concentrated in computer or media labs; or of the teachers' unawareness of available teaching resources, and the need of having to adapt them to their classroom situation, which is often essential for their use; or of the insufficient theoretical and practical training that teachers have to allow them to integrate conceptual frameworks into learning environments with computer support.

Our research in recent years has sought to contribute computer-based environments for mathematics teaching and learning. For example, the current development of the intelligent tutorial system *GeogebraTUTOR*, which concerns us in this article, is based on research and experiences with the previous system *AgentGeom* (Cobo & Fortuny, 2007). In the new system we have kept the style and structure of the existing student system communication of *AgentGeom* and we have integrated the open source software *GeoGebra* (Hohenwarter & Preiner, 2007) as part of its interactive user interface. With this addition our intention is to provide instrumental learning of the skills of dynamic geometry. In addition to improving the students' user interface, and adapting it to formats more in accordance with those that students frequently use in their everyday virtual communications (like chats), the new system considerably eases their communication with the teacher, making it very easy to implement new problems and message systems.

Our aim in using the *GeogebraTUTOR* tutorial system is to support students to develop mathematical skills relating to problem solving, mathematical reasoning and communication processes with the help of mathematical language, either by facilitating dialogues with the tutorial system, which is considered a cognitive system, or by using the processes of dynamic geometry favoured by the system (representation, visualisation, demonstration, etc.).

In the *GeogebraTUTOR* (or *AgentGeom*) environment, our research has been organised around teaching, ways to improve students' mathematical skills, and technology, in an attempt to integrate new technologies into teaching and learning. This research is beginning to show the need for methodological teacher training that considers both teaching and technology in order to continue contributing to the use of computers in mathematics classrooms.

When it comes to addressing initial or permanent teacher training, trainers must respond to questions such as the following: What type of professional knowledge and skills should trainee teachers acquire? In what area of practice and by means of which methods? How can technology help to facilitate such professional development?

In this paper we will attempt to provide an answer to these questions and to situate the technological medium in teacher training. To do this, we define the professional teaching skills and the processes of learning required to teach mathematics within the context of using

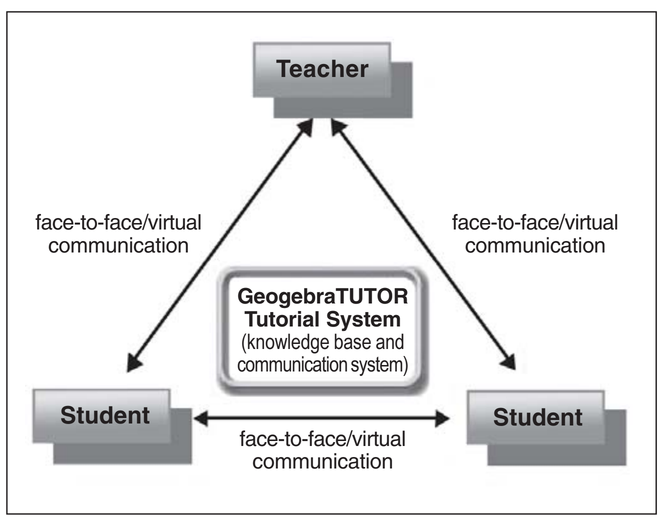

Figure 1: Paradigm Technology

the computer in the classroom; and we describe the context in which the experience is developed, providing examples of activities for initial teacher training which integrate the teaching of problem solving into the *GeogebraTUTOR* environment, highlighting references to our theoretical framework and analysing the methodology which combines the use of the tutorial systems with face-to-face teaching models.

2 DEFINING THE KNOWLEDGE NECESSARY TO TEACH AND THE PROCESSES OF LEARNING TO TEACH MATHEMATICS

2.1 The technological paradigm

The introduction of the computer and its increasing influence on the processes of teaching and learning have contributed to improvement of the paradigm that represented the insertion of technological media only at the time of their application in the classroom. As a consequence of this change, the approach to using new technologies finds a new place in curriculum planning, the design of lesson plans and teaching itself. In this respect, in order to be able to plan training for teachers who have computers available to them as a support we must stipulate, on the one hand, their roles with regard to the students, the technological medium and the transfer of knowledge, and on the other, our ideas regarding the professional skills of mathematics teachers in these new situations and the processes for acquiring and generating these skills. All of the above will provide the foundation for our proposal for initial teacher training.

The teacher's traditional role as a mediator of basic knowledge and interactions between students is gradually being substituted by teaching and learning processes where communication among students and between the students and the teacher is always bi-directional. As can be observed in Figure 1, in these processes the students interact with the technological medium and learn directly from it. They interact among themselves and with the teacher by means of communication systems which are virtual (visual) or face-to-face (oral or visual). In this way, teaching is moving away from a focus on the classroom and the teacher as the transmitter of knowledge towards the technological medium and the student. The teacher is becoming a person who supervises and manages all phases of the teaching and learning process, in what has become known as the «technological paradigm» (Branson, 1990).

Specifically, in this technological paradigm the teacher will be in charge of:

- Analysing mathematical teaching proposals implicit in the technological medium and proposing modifications of the new plans and ways of organising content, if deemed necessary.
- Managing face-to-face or «live» mathematical communication, whether in the classroom or virtually via an online discussion forum.
- Analysing, diagnosing and bringing meaning to students' work; also, collaborating with the tutorial system on student evaluation.

2.2 Professional skills of the mathematics teacher

Llinares (2002) states five components characteristic of the professional knowledge of the mathematics teacher: knowledge of and about mathematics; knowledge of the curriculum; knowledge of learners' cognitions; pedagogic knowledge specific to mathematics; and knowledge about its teaching. In accordance with the role of the mathematics teacher within this new technological paradigm, we must add to these domains of the teacher's professional knowledge specific knowledge regarding the technological medium and, in particular, the computer. According to Badía (2006), we can specify this component of knowledge still further: knowledge of technological characteristics with clear teaching implications.

In this paper we will discuss the professional skills of teachers in the sense of abilities to use knowledge and in contexts and situations related to the processes of teaching and learning problem solving with computer support. Therefore, the trainee teacher must have professional knowledge and be capable of putting it into practice in an effective way.

By integrating Llinares' definition of professional knowledge and Badía's technological component we therefore propose five professional skills for teachers in the context we are considering: heuristic, curricular, discursive-instrumental, semiotic, and teaching skills, which we are going to analyse in section 2.3.

Our proposal is in line with integrating and transforming knowledge into practice in a coherent and systematic way, and as a consequence we will tend to develop cognitive actions such as: observing, predicting, criticising, generating and analysing (LLINARES, 2002). Following these principles, we will propose activities and take advantage of the debates arising from these to situate the knowledge generated within the corresponding theoretical frameworks (see section 4).

To be specific, the initial teacher training we propose will be aimed at studying the learning and generating of knowledge necessary for teaching mathematical problem solving in an interactive tutoring system called *Geogebra TUTOR*, for learning mathematical geometry skills in secondary education. In this respect, our research team has already developed a prototype tutorial system, called *AgentGeom*, which allows the user, whether expert or teacher, to introduce the heuristic and discursive characteristics of a problem that students should solve in the context of a given didactic contract.

The term *GeogebraTUTOR* refers to both a tutorial system (TUTOR) and an emerging approach for instrumental learning of skills involved in dynamic geometry (GeoGebra). Thus our project contributes an original approach to research on teaching based on the idea of a reciprocal adaptation process between the development of the tool – our tutorial system as a technological instrument – and its use as a means of instrumentation in teaching situations.

In teacher training we consider two didactic relationships: the real relationship «teacher (instructor) - student (future teacher) - mathematics teaching», and the simulated relationship «teacher-student-mathematics». The teacher tends to have a dual role, that is, a real role (of an «instructor») and a simulated role (of a «teacher»), although he or she is not accustomed to simulating the role of a student (as there are no technological means to this end), or controlling his or her role as a teacher (simulated) in the role of an instructor (real). Our technological instrument (*GeogebraTUTOR*) allows the simulation of the «teacher-student-mathematics» relationship in teacher training.

Figure 2 provides an overview of our objectives in proposing initial teacher training to problem solving in the *GeogebraTUTOR* environment, relating the roles of the typical behaviour of the «solving» teacher to his or her respective professional tasks, in line with the ideas expressed by Callejo (2006).

In this initial training proposal, we would like to highlight the significance of reflecting on the actions that are being carried out – hence the adjective «reflective» used to describe the roles of solver and teacher in the training process.

With "reflecting on the action" we mean the promotion of elements of reflections among trainee teachers which contribute to developing their learning processes. Schön (1987) identifies five fundamental elements that facilitate these reflections which we have attempted to include in our proposal: technological-student-teacher interaction must itself become the object of reflection; the description of knowledge itself must be promoted; there must be reflection on how to understand one another; making use of one's own and others' experiences must also become an object of study; and the theories implicit in communicative processes must be highlighted.

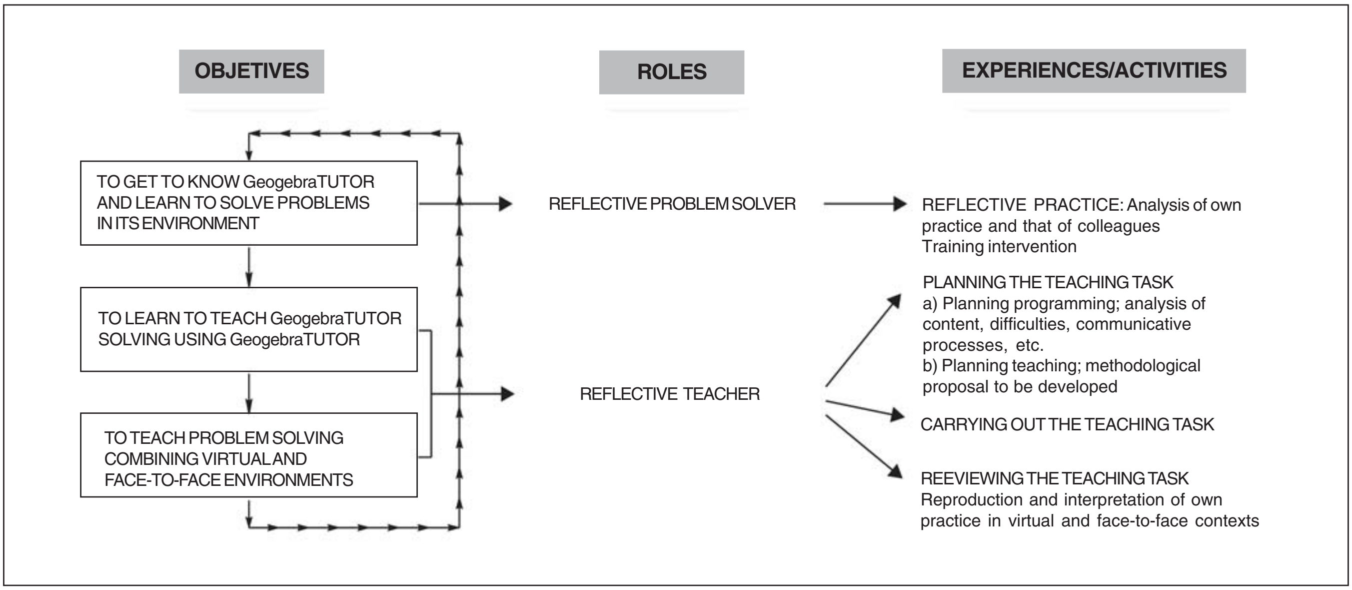

Figure 2: An overview of our objectives

2.3 Professional skills in managing problem-solving classes with computer support

For the management of problem-solving classes we have considered that the teacher must develop professional skills relating to heuristics, the curriculum, the discursive-instrumental relationship, semiotics, and didactic.

2.3.1 Heuristic skills

We give heuristic skills a broad meaning which includes: heuristics, ways of focusing solving, argumentative processes, controlling and managing elements of solving processes, etc. In short, the analysis and putting into practice of constructive models for teaching and learning of problem solving.

2.3.2 Curricular skills

Other mathematical content (conceptual and procedural) involved in problem solving and its place on the mathematics curriculum. For this aspect we introduce concepts such as the basic space of a problem, problem tree and problem forest (Richard & al., 2008).

2.3.3 Discursive-instrumental skills

The discursive-instrumental skill includes analysing students' interactions with both the technological medium (simulated social debate), and with the teacher in carrying out the tasks proposed. At the beginning of this analysis, we introduce the notion of the basic space of the tutorial action, which forms the basis of the discursive component we have implemented in the *GeogebraTUTOR* tutorial system (Cobo & al. 2007).

The recent development of tutorial systems, and in particular our prototype *AgentGeom*, allows the establishment of a relationship between the student and the system in which the different virtual pedagogic agents work together to resolve the same situation-problem. In order to resolve a given problem, students are being provided an interface to construct an element of a figure or to write a discursive proposition to introduce a calculation or a structured inference. If necessary, the system responds to the student's actions in real-time with the help of written messages. These messages are prepared in advance by the teacher according to the set of possible inferences within the logic established by the basic problem space. For the teacher or researcher it is possible to adapt the inferences and its associated messages to the characteristics of a specific didactic contract, such as, for example, one which allows contradiction as an integral part of the solving process or one which accepts the use of graphical propositions in the reasoning structure.

Furthermore, in the analysis of interactions between students and technological medium it must also be taken into account that, according to Rabardel (1995 & al, 2001), the computer program influences «not only the user's manner of acting, but also their way of thinking», and that the student needs to consciously mobilise knowledge control structures which are controlled during the «instrumental genesis». All of this occurs despite the linking of technical components in the students' development of mental schemas, or students only having indirect access to the mathematical models underlying the construction of the program – either symptomatically or confirming the effect of the action-reaction mechanism, in accordance with Rabardel's theory of instrumentation.

2.3.4 Semiotic skills

In a learning situation, the question of semiotics is often underestimated by mathematicians or computer experts. Furthermore, it is also often ignored in the contents of teacher training programmes. We believe that the future teacher must be capable of analysing the influence of using different semiotic representation systems in students' cognitive development, based on the ideas we expose below.

Any semiotic representation system, whether figural signs, mathematical symbols or word symbols in a natural language, possesses both producing and reducing effects in the representation of knowledge. That is, if the mobilised signs allow certain properties to be «seen», they also prevent others from being «seen». This paradox is a determining factor in the acquisition of knowledge, whether on a level of communication, the treatment of cognitive representations or the objectivation of virtual representations (those in the mind of the student, for example).

Geometry is characterised by the use of figures. Nowadays, it is normal to insist on different forms of viewing to those that may be used in mathematics in order to make concepts more accessible. Visualisation is considered a more economical support or medium for comprehension. By introducing the notions of graphic reasoning and figural inference (Richard, 2004a, b), we have shown that all modes of visualisation that can be used in mathematics constitute registers for treatment (processing) which allow autonomous rhythms of thought as powerful as the different discursive registers (verbal and symbolic). Thus, as a geometrical design links reasoning steps, it is natural for the student to integrate their figural representations into the discursive structure of their reasoning. But the main interest in these notions is perhaps found in their distinction. While the notion of graphic reasoning sends a mathematical treatment in a unique register, figural inference can be combined with semantic and discursive inferences (in the sense of Duval, 1995) within the unit of a single thought.

In summary, the figural inference allows the discovery of a transversal reasoning with diverse registers. Therefore, in problem solving and in a situation of validation, the social debate must integrate the coordination of diverse representation systems (figural, verbal and symbolic) during the progression of reasoning and, of course, dialectic argumentations.

2.3.5 Didactic skills

The teaching skill is not only related to the ability the teacher has in enabling the students to achieve the skills they are aiming for, but also in developing their autonomy. From a constructivist perspective, the effective construction of pertinent knowledge is achieved by changing the students' conception and overcoming obstacles such as situations that reproduce the constitutive characteristics of mathematical work. According to Brousseau (1998, p. 61), the only way to «do» mathematics is to look for and resolve certain specific problems and, to this end, approach new questions. If this is the case, teaching situations are paradoxical in two ways. Firstly, the teacher wants the student to produce appropriate responses, thereby displaying the, at least apparent, success of the teacher. However, the student does not have the necessary cognitive or discursive means: it is precisely the job of the teacher to make it possible for them to have these. Secondly, it is considered that the student will not have acquired full knowledge until he or she knows how to put it into practice autonomously, without the help of the teacher or his or her colleagues.

To minimise these paradoxes, the trainee teacher must be aware that social debate appears as a medium that guarantees students to work on the same situation-problems as those proposed and to respect its internal logic and indispensable conditions for developing autonomy. As a consequence of the above, we can say that the theoretical context of our proposal for initial teacher training may be situated in a hyperspace in which professional skills – heuristic, curricular, discursive-instrumental, semiotic and teaching skills – are constituted as the bases to achieve the professional and technological objectives as expressed in section 2.2 of this paper.

3 CHARACTERISTICS OF THE CONTEXT FOR THE TEACHER TRAINING EXPERIENCE

In an attempt to specify the field of practice in which trainee teachers are to acquire knowledge, in this section we shall determine the general characteristics of the students who participate in the experience and our conception of how to focus teaching in the context of teaching practice. In the following paragraph, we provide examples of activities for initial teacher training in the computer science environment of *GeogebraTUTOR*.

This experience will take place within the initial training programme for secondary school teachers and, specifically, in teaching practice for the *Curs de Qualificació Pedagògica* «Pedagogical Qualification Course» (CQP) taught at the UAB's Department of Didactics of Mathematics, and at the *Centre de formation initial des maîtres de l'Université de Montréal* to students who have just graduated in Mathematics or in other Sciences (Physics, Computer Science, Engineering, etc.), thereby guaranteeing, in general, a good mathematical background but lack of teacher training.

With regard to specifying our way of conceiving teaching practice in initial teacher training, we agree with Schön (1987) in considering it a type of reflection-in-action in which there is analysis of typical practical situations for students or situations proposed by the tutor in which it cannot be assumed «either that the existing professional knowledge adapts itself to each case or that each problem has a correct response» (p. 47). Therefore, trainee teachers must not be satisfied with simply applying known facts, rules and procedures to instrumental problems in a non-conflictive way, nor even learn to think like the tutor leading the practice sessions, although at specific times this may help them, but rather that their reflection-in-action must also include «the construction and verification of new categories of knowledge, strategies of action and ways of formulating problems» (p. 47), it being, in these cases, the role of the tutor to promote reflections that tend to achieve these objectives.

In our technological context the use of the GeogebraTUTOR system allows simulation of the whole relationship «teacher-student-mathematics» with a dual role for teachers, facilitating their professional development. Normally, what teachers find most difficult is acquiring sufficient self-confidence to be able to ensure control of their tutorial action (from a socio-constructivist perspective) and foster the students' development of autonomy.

4 EXAMPLES OF ACTIVITIES FOR THE INITIAL TRAINING OF TEACHERS IN THE COMPUTER SCIENCE ENVIRONMENT OF GeogebraTUTOR

In this section we will be simultaneously introducing the working methodology and the contents of the activities we propose. We shall therefore specify the methods used to achieve our objectives. Furthermore, we will present the activities by matching them with the proposed objectives (see Figure 2).

4.1 Enhancing knowledge of GeogebraTUTOR and learning to resolve problems in its environment

First of all, trainee teachers are presented with the basic characteristics of the *GeogebraTUTOR* system – interfaces, forms of communication, etc. – and they are immediately assigned a given problem from the system, in order for them to familiarise themselves with its use. In this initial stage, therefore, familiarisation with the system is combined with beginning to learn how to resolve problems in this environment. Furthermore, teachers will be asked to complete the problem solving report – provided by the system – in as much detail as possible, taking notes about the moments they consider to be vital in the problems they have solved and their thought processes. Then, they have to attempt to solve the same problem in different ways.

Trainee teachers are asked to reflect successively on the following questions in the *GeogebraTUTOR* virtual debate forum:

- Analyse the uses of the graphic and deductive areas that you have implemented throughout each problem-solving process. How has the introduction of mathematical knowledge evolved with the different uses of the aforementioned work areas?
- Analyse the proactive and reactive nature of your interventions.
- What semiotic representation systems do you think the system has provided you with in terms of mathematical content?
- Study the nature of the mathematical information the system displayed in the messages throughout the solving processes and the influence these have had in the development of said processes.
- Considering the reflections generated in the above sections, analyse one of the solving processes using Schoenfeld's categories (1985) or others you define yourself.
- Using the solutions you have found or contributions of your colleagues, attempt to construct a graph that illustrates all of the solutions found that resolve the problem proposed.
- Reflect on the knowledge learned as a consequence of your interaction with the *GeogebraTUTOR* tutorial system and post your reflections on the virtual discussion forum.

4.2 Formative intervention

When managing the virtual discussions that originate as a consequence of the above reflections, the tutor suggests to trainee teachers readings of theoretical works referring to knowledge arising in the dialogues, in accordance with the following subject matter:

- Interactions between the teacher-student-technological medium as an object of reflection. Characteristics of the tutorial system discourse. Message systems (COBO & FORTUNY; 2000, 2007).
- Integration of different semiotic representation systems. Semiotic axis of the theoretical foundation of *GeogebraTUTOR* (RICHARD, & al. 2007).
- Some aspects relating to the theory of instrumentalization. Technological axis of the theoretical foundation of *Geogebra TUTOR* (RICHARD, & al. 2007).
- Adapting the *GeogebraTUTOR* tutorial system to the cognitive characteristics of students (COBO & FORTUNY, 2007).
- Generating the *GeogebraTUTOR* knowledge base. Introduction to the «basic space of a problem» (COBO, 2004a).
- Polya's ideal problem-solving model and Schoenfeld's knowledge components and behaviour for explaining students' actions when resolving mathematical problems (model for action) (POLYA, 1975; SCHOENFELD, 1985).
- Situating the proposed problem on the mathematics curriculum and modes of considering problem solving on the curriculum (CALLEJO, 1999).

- Establishing a model to analyse the processes you have developed for resolving problems considering the above reflections and contributions (Cobo, 2004a).

Following the virtual debates, there are face-to-face sessions for trainee teachers to present summaries of their virtual interventions and discuss these whilst completing and summarising the theoretical foundations of the *GeogebraTUTOR* tutorial system and the solving of problems and aspects of its operation.

4.3 Learning to teach problem solving in the GeogebraTUTOR environment

The activities we propose for achieving this objective can be carried out individually whilst expounding reflections, questions, contributions, etc., on the virtual discussion forum. At the end of the activities, as with the above section, there are face-to-face sessions for in-depth analysis of the most important aspects of each one. Activities have the following structure: trainees select one of the tutorial system's problems and attempt to construct its basic space. They then use this to reflect on the following training activities.

- **Problem solving analysis:** Attempt to state the mathematical content involved in the possible solutions. Identify the key points of each branch of the basic space of the problem. Propose a message system, on different levels, that the system should send to the students in the event that they get stuck on one of these key points. Using the basic space and the mathematical content necessary for solving it, attempt to modify the original principle to generate simpler problems (on lower levels) and more difficult problems (on higher levels). Propose others with different principles which involve similar content and different levels of difficulty. How do you think the system favours these processes?
- **Identifying the formulation of conjectures and tests for these:** Try to predict what type of difficulties you think students will have in resolving the problem. Evaluate the progression of cognitive and heuristic skills in a process of collaborative solving. Also identify whether the interlocutors are reasoning about the same objects, contributing to training or to the testing of the same conjecture and whether responsibility for the initiatives or for the reactions are truly shared in the argumentative process, that this, whether a cooperative interaction is really established.
- **Analysing the level of learning and the level to which students have developed their autonomy:** How do you think *GeogebraTUTOR* should act when faced with the difficulties of the previous section (what type of messages should it send?). Evaluate the level of learning (Moschkovich, 2004) from a socio-cultural perspective to describe learning through interaction, which positions the discursive activity at the centre of learning mathematics (Forman, 1996, Lerman, 2001). Let us consider that the student's search for autonomy should be situated in relation to the acquisition of knowledge (facts, discursive or cognitive means). However, the evaluation of said acquisition only makes sense after a long period of learning and in relation to various situation-problems that comprise the same concept (Vergnaud, 1999). Students are autonomous if they themselves are able to construct the new knowledge (conclusion) from the conditions (hypothesis) of the situation-problem, which necessarily involves cognitive progress.
- **Indicators for evaluation:** Identify the indicators the *GeogebraTUTOR* interactive tutorial system uses to evaluate students, using labels (markers) for concepts, heuristics, semiotic aspects and mathematical processes – which also allow the problems to be defined. Markers are identified by the system by means of the student's graphic and discursive actions. Reflect on their importance in the context of the student's problem solving in order to generate a qualitative evaluation of the

mathematical skills of the students. When one of the students has resolved the problem, the initial characterisation becomes an oriented graph that retains the order in which the labels were stored in the solving process. Reflect on and argue the possibility of including other indicators in addition to those contemplated by the system. Complete the evaluation report the tutorial system presents to the student (REVERTER & al., 2007).

4.4 Reproducing and reinterpreting own practice in contexts that combine virtual and face-to-face environments

This phase of initial teacher training must be based on a fundamental principle, such as considering the teacher as a reflective individual who must reproduce and reinterpret his or her own practice. It is for this reason that not only reflective processes must always be present, but also interpretation and revision of the teaching process, as we highlight in this section.

In the context in which the teaching practice is carried out, teachers need to do teaching practice of 6 to 8 hours with students, planning and experimenting with *GeogebraTUTOR* as a supporting tool, before reviewing and reproducing.

4.4.1 Planning programming

During the planning phase, teachers may choose between: selecting one of the problem itineraries already implemented in the tutorial system; programming a new itinerary based on the proposal and implementation of interlinking problems over which the system is able to move in order to adapt to the cognitive characteristics of the students; or following a process somewhere between the two, by means of which one of the itineraries implemented in the system is selected and only some of the problems in it are modified. In all cases, during this planning phase teachers must:

- Analyse the cognitive and social characteristics of the group-class with which you are working, in order to be able to initially assign problems from the system according to those characteristics.
- Justify the hierarchical positioning of problems within the itinerary on the basis of the problem-solving strategies each of these requires for it to be solved (basic space of the problem) and establish pedagogical criteria which allow the system to move within the problem tree in order to adapt itself at all times to the cognitive characteristics of the students.
- Taking as a reference the cognitive and social characteristics of the students involved in the practice, justify the choice of message system to be implemented in *GeogebraTUTOR*, or propose modifications to those already implemented, depending on which option you have chosen, using the set of messages proposed by Cobo (2004a) or others the teacher considers appropriate (construction of the basic space of the tutorial action).
- Decide the position on the curriculum for the mathematical content of the problems considered (mathematical skills that contribute to developing, concepts, heuristics, forms of arguing that favour, types of languages and systems of representation that facilitate, etc.).
- Predict indicators to complete the evaluation reports the *GeogebraTUTOR* system is to issue to each student.

4.4.2 Planning teaching

Once the teaching experience with the students has been planned with regard to content, teachers must plan the methodology they will follow during teaching practice. In this respect, they must reflect on how to develop the experience:

- Face-to-face and/or virtual sessions;
- Content of each;
- Forms of interaction (managing the teaching and learning process);

- Idea-sharing sessions and means of managing them (COBO, 2004b);
- Prevision of new knowledge to introduce in said sessions, etc.

4.4.3 Reflection, revision and synthesis

Once practice with students has been completed, the plan is for a face-to-face idea-sharing session for the more relevant results each teacher has encountered. In this session, the advantages and difficulties of each stage of the teaching practice will be analysed and, in particular, the experience of having developed the process of teaching and learning problem solving in an ICT environment like *Geogebra TUTOR*. A further aim of this session is to formulate open questions with regard to this process in the manner in which they are formulated in the experience developed by Callejo (1999).

4.4.4 Compiling a report

To complete this initial teacher training, participants must prepare a report which comprises the three stages of their initial training process with *GeogebraTUTOR* – planning, developing and reviewing the teaching task – and in which they give priority to the reflective and critical elements over the purely descriptive ones. In view of the results of the teaching practice with students and following the final session of the face-to-face debate, trainee teachers are asked to reformulate their initial plan. By means of a guideline, below we state just some of the aspects they are invited to reflect upon when reformulating their plan:

- How have you managed interactions in class? Do you think this could be improved? How?
- Have you provided students with special multimedia or hypermedia resources? Which and for what purpose? Review these and make proposals for improvement.
- Analyse the students' problem-solving processes and justify in them the itineraries the system has proposed to the students within each problem tree and forest. Justify these itineraries in terms of what students have learned with reference to their initial knowledge. Do you think that the allocation was correct for each case? Make justified proposals in your practice which improve the system's allocation of itineraries.
- Analyse and review the criteria the system uses to evaluate students and those you have contributed to complement this evaluation. Would you change any of them? Justify your decisions.

5 CONCLUSIONS

In this paper we have laid out a proposal for the initial training of mathematics teachers in secondary schools, using the intelligent tutorial system *GeogebraTUTOR* as a reference point on both a theoretical (based on the educational theories behind the system) and practical level (analysing proposals developed in its environment), that is, in the context of the technological paradigm.

In the proposal we have considered five professional skills for teachers: heuristic, curricular, discursive-instrumental, semiotic, and teaching skills as the basis for initial training of teachers who teach problem solving at secondary school level.

We have highlighted the importance we give to training based on reflection-in-action in all stages of the development of the teaching task, including: the interactions produced in the context in which the proposal is developed; description of and reflection on own and others' knowledge; and reference to the theories involved in communicative processes, as shown in the activities we propose for the four training phases: i) enhancing knowledge of *GeogebraTUTOR* and learning to resolve problems in its environment, ii) formative intervention, iii) learning to teach problem solving in the *GeogebraTUTOR* environment and iv) reproducing and reinterpreting own practice in contexts that combine virtual and face-to-face environments.

The approach we have formulated in this paper is framed in our future research project: «geogebraTUTOR: a new approach to research into interactive skills learning for geometry on a secondary level» (ARIE-2007, CRSH-2007), with the following objectives:

Objective 1 (Instructional model)	To analyse the teaching effectiveness of an interactive tutorial system (*geogebraTUTOR*) in terms of improving the development of mathematical skills by means of demonstration problem solving for secondary students.
Objective 2 (Interpretation and theorization)	To interpret and theorize on representational, teaching and instrumental characteristics with regard to developing student autonomy in an interactive tutorial system.
Objective 3 (Evaluation and control)	To evaluate the evolution of the subject-medium system in interaction with the interactive tutorial system with regard to control structures for semiotic and cognitive conditions.

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